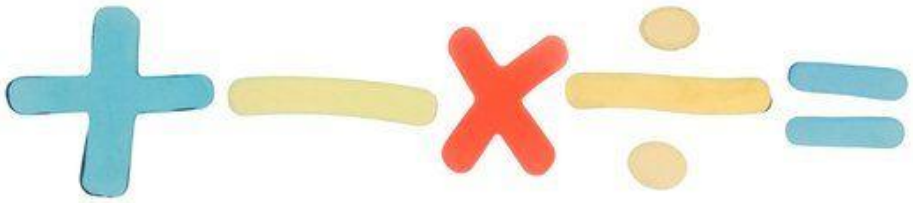
Tetherdown School

Progression in Calculations



Introduction

Rationale

At Tetherdown Primary School, children show high standards in mathematics. This document reinforces the progress in mathematics to ensure that children have a secure understanding of standard written methods by upper Key Stage 2.

This policy provides an outline of which mental and written strategies are to be taught and in which year group, so that in Year 6, they will only need to review standard written methods rather than beginning to learn them.

The teaching staff worked together, in year groups to agree the progression outlined in this policy.

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

Written Calculation Strategies

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording.

Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation. Whatever method is chosen (in any year group), it must still be underpinned by a secure and appropriate knowledge of number facts and place value.

Strategies for calculation need to be supported by familiar models and images to reinforce understanding (See Appendix A). When teaching a new strategy it is important to start with numbers that children can easily manipulate so that they can understand the concept.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time. Therefore, this policy is a guide to what can be achieved at Tetherdown given high standards. However, staff must be aware that previous stages may need to be revisited to consolidate understanding when introducing a new strategy. The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.
Children need to acquire one efficient written method of calculation for each of the four operations which they know they can rely on when mental methods are not appropriate.

March 2013

Written methods of calculation: addition and subtraction

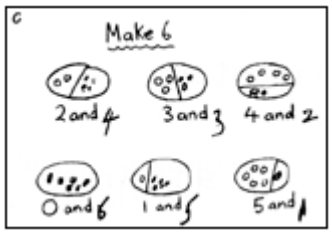
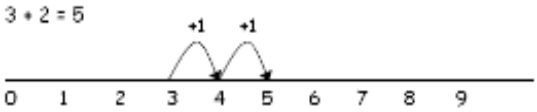
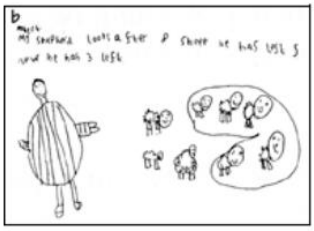
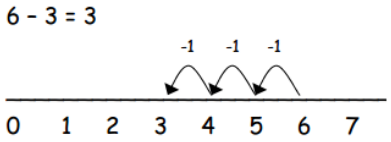
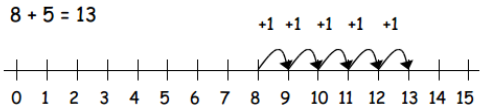
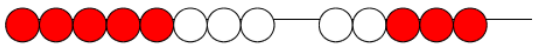
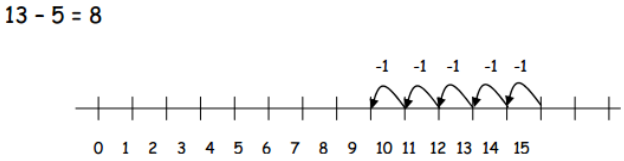
WHEN ARE CHILDREN READY FOR WRITTEN CALCULATIONS?

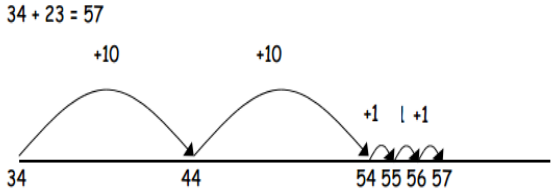
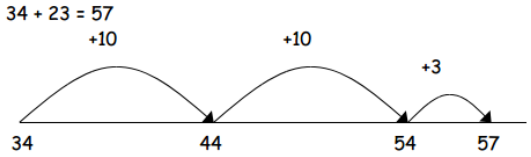
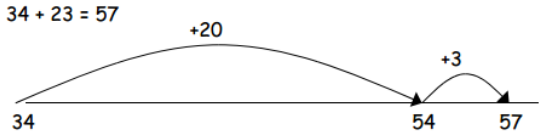
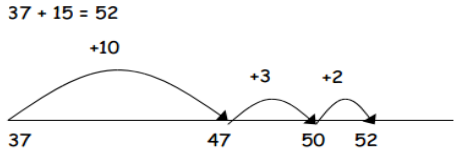
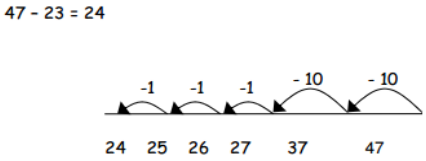
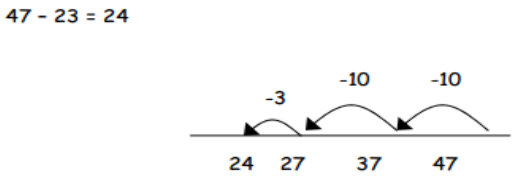
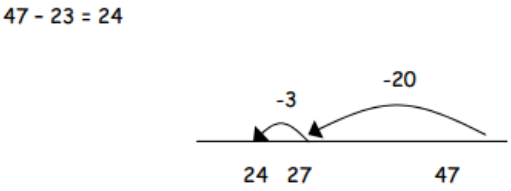
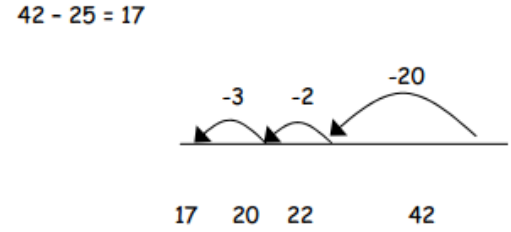
To **add** successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

To **subtract** successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 - 70) using the related subtraction fact, 16 - 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

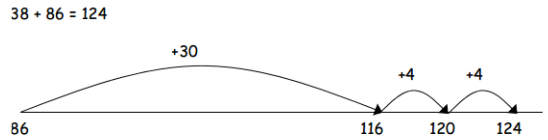
	Addition	Subtraction
Reception	Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.  They use numberlines and practical resources to support calculation and teachers demonstrate the use of the numberline. 	Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.  They use number lines and practical resources to support calculation. Teachers demonstrate the use of the number line.  The numberline should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.
Year 1	Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.  Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3. 	Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.  Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2. 

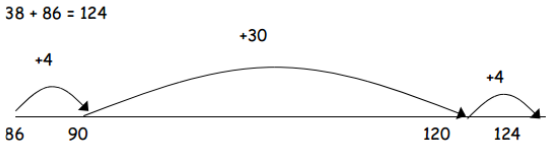
Year 2	Children will begin to use empty number lines themselves starting with the larger number and counting on. • First counting on in tens and ones.  • Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).  • Followed by adding the tens in one jump and the units in one jump.  • Bridging through ten can help children become more efficient.  Partitioning This process should be demonstrated using base 10 and arrow cards to show the partitioning. 67 + 24 60 + 20 = 80 7 + 4 = 11 80 + 11 = 91	Children will begin to use empty number lines to support calculations. Counting back • First counting back in tens and ones.  • Then helping children to become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4).  • Subtracting the tens in one jump and the units in one jump.  • Bridging through ten can help children become more efficient.  Begin to use counting on.

Year 3

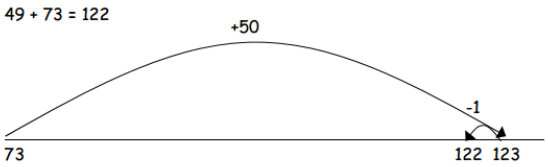
Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

- Count on from the largest number irrespective of the order of the calculation.





- Compensation



Column addition

Introduction to column addition, using partitioning

67 = 60 + 7

24 = 20 + 4

80 + 11 = 91

67
+ 24
80 (60 + 20)
11 (7 + 4)
91

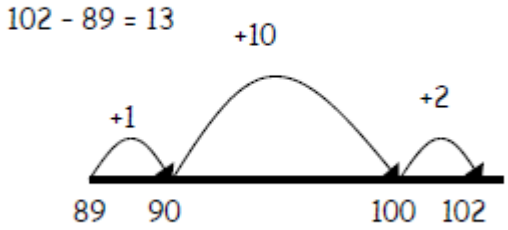
Adding the least significant digits first (in preparation for carrying)

67
+ 24
11 (7 + 4)
80 (60 + 20)
91

Column addition with carrying, with up to 3 digits.

Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.



Children will continue to use empty number lines with increasingly large numbers.

Column subtraction without borrowing

48
- 16
32

Column subtraction with borrowing

5 13 1
6467
- 2684
3783

Year 4	Column addition with numbers up to 1 decimal place and carrying. 625 + 48 = 673 (1) 783 + 42 = 825 (1) Children should extend the carrying method with up to 4 digits. Children should: • add several numbers with different numbers of digits; • begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds; • know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.	Column subtraction with numbers up to 1 decimal place and borrowing. 5 13 1 6467 - 2684 3783 Children should: • using this method, children should also begin to find the difference between two or three-digit sums of money, with or without 'adjustment' from the pence to the pounds; • know that decimal points should line up under each other.
Year 5	Column addition with numbers up to 2 decimal places 6584 + 5848 = 12432 (1 1 1) 7648 + 1486 = 9134 (1 1 1) Children should extend the carrying method with up to 5 digits. Children should: • add several numbers with different numbers of digits; • begin to add two or more decimal fractions with up to four digits and either one or two decimal places; • know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.7	Column subtraction with borrowing with numbers up to 2 decimal places 5 13 1 6467 - 2684 3783 Children should: • be able to subtract numbers with different numbers of digits; • begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places; • know that decimal points should line up under each other
Year6	Children should extend the carrying method to number with any number of digits.	Children should extend the borrowing method to number with any number of digits.

Written methods of calculation: multiplication and division

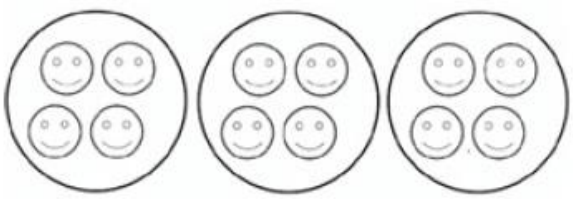
It is important that children's mental methods of calculation are practised and secured continually, alongside their learning and use of an efficient written method for multiplication. These notes show the stages in building up to long multiplication and long division through Years 4 to 6.

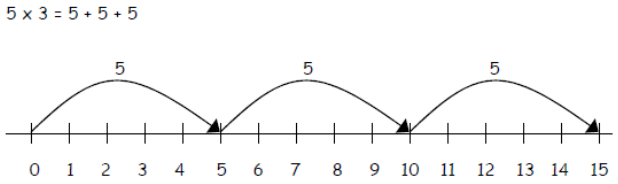
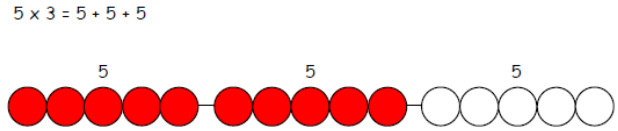
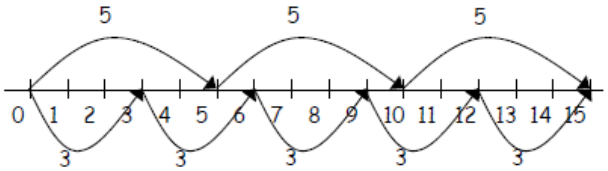
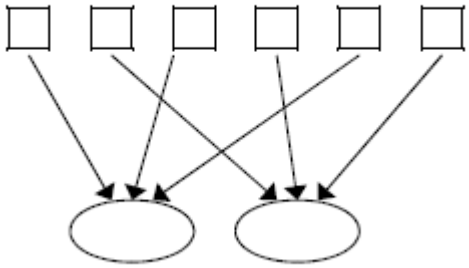
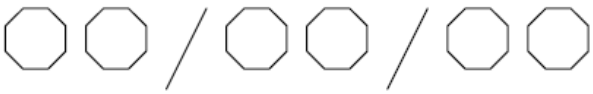
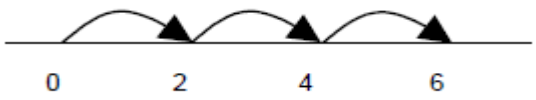
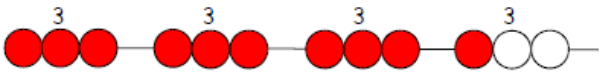
Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

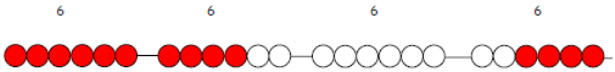
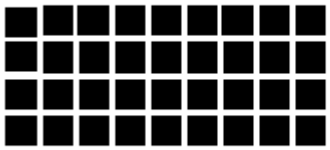
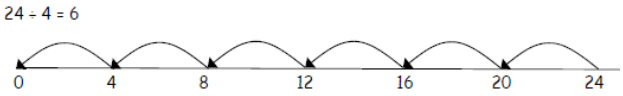
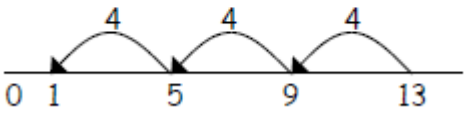
To **multiply** successfully, children need to be able to:
• recall all multiplication facts to 12 × 12 (by the end of Year 4);
Children will learn the following times tables:
Year 2: 2x, 5x, 10x
Year 3: 2x, 3x, 4x, 5x, 6x, 8x, 10x
Year 4-6: all multiplication facts to 12 × 12
• partition number into multiples of one hundred, ten and one;
• work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
• add two or more single-digit numbers mentally;
• add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
• add combinations of whole numbers using the column method

To **divide** successfully in their heads, children need to be able to:
• understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
• partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
• recall multiplication and division facts to 12 × 12, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
• know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
• understand and use multiplication and division as inverse operations.

To carry out **written methods of division** successful, children also need to be able to:
• understand division as repeated subtraction;
• estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
• multiply a two-digit number by a single-digit number mentally;
• subtract numbers using the column method.

	Multiplication	Division
Reception and Year 1	Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups. 	Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s. 

Year 2	Children will develop their understanding of multiplication and use jottings to support calculation: • Repeated addition 3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3 Repeated addition can be shown easily on a number line or a bead bar   • Commutativity Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.  • Arrays Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.  5 x 3 = 15 3 x 5 = 15	Children will develop their understanding of division and use jottings to support calculation: • Sharing equally 6 sweets shared between 2 people, how many do they each get?  • Grouping or repeated subtraction There are 6 sweets, how many people can have 2 sweets each?   • Repeated subtraction using a number line or bead bar 

Year 3	Children will continue to develop their understanding of multiplication and use jottings to support calculation: • Repeated addition 4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4   • Commutativity • Arrays  9 x 4 = 36 9 x 4 = 36 • Partitioning 38 x 5 = (30 x 5) + (8 x 5) = 150 + 40 = 190 • Grid method TU x U 23 x 7 = 161 T U x 20 3 7 140 21 140 + 21 = 161	Ensure that the emphasis in Y3 is on grouping rather than sharing. Children will continue to use: • Repeated subtraction using a number line Children will use an empty number line to support their calculation.  Children should also move onto calculations involving remainders. 13 ÷ 4 = 3 r 1 

Year 4

- Grid method

TU x U

23 x 7 = 161

x	T: 20	U: 3
7	140	21

140 + 21 = 161

HTU x U

123 x 3 = 369

X	H: 100	T: 20	U: 3
3	300	60	9

300 + 60 + 9 = 369

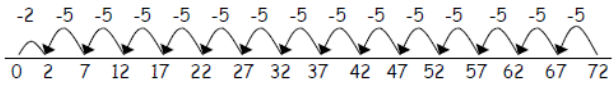
TU X TU

38 x 72

x	70	2	
30	2100	60	= 2160
8	560	16	= 576 +
			2736
			1

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

72 ÷ 5



- Short division- division by a single digit

96 ÷ 6 = 16

$$6\overline{)936}\quad 16$$

196 ÷ 6 = 32 r 4

$$6\overline{)1916}\quad 32\ r\ 4$$

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.
Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example, 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.
e.g. I have 62p. Sweets are 8p each. How many can I buy? Answer: 7 (the remaining 6p is not enough to buy another sweet)
Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed? Answer: 8 (the remaining 6 apples still need to be placed into a box)

Year 5

Children continue with grid method with more digits

TU x TU
38 x 72

x	70	2	
30	2100	60	= 2160
8	560	16	= 576 +
			2736
			1

ThHTU x U

1125 x 7 = 7875

X	Th	H	T	U
	1000	100	20	5
7	7000	700	140	35

- Grid method for decimals

7.2 x 3.8

x	7	0.2	
3	21	0.6	= 21.60
0.8	5.6	0.16	= 5.76 +
			27.36
			1

- Short multiplication – multiplication by a single digit

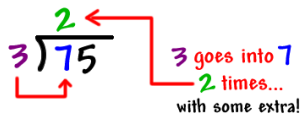
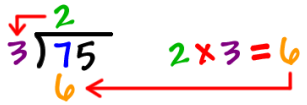
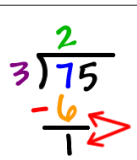
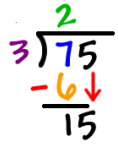
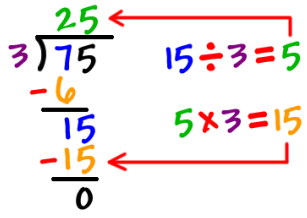
```
  346
x   9
 54  (6 x 9)
360  (40 x 9)
2700 (300 x 9)
3114
```

Leading to

```
  346
x   9
3114
4 5
```

- Children will continue to use short division for TU ÷ U, HTU ÷ U, ThHTU ÷ U
- Extend to division of decimal numbers by a single digit, up to two decimal places
- Children begin to use long division using the mnemonic Daddy, Mommy, Sister, Brother (Divide, Multiply, Subtract, Bring down)

```
  54
9)486
 -45
  36
 -36
   0
```

Divide:	
Multiply:	
Subtract:	
Bring Down:	
Repeat:	

Year 6	• Long multiplication- multiplication by a double digit Children should multiply numbers with at least 4 digits by a 2 digit whole number. 352 × 27 2464 (352 × 7) 7040 (352 × 20) 9504 1	• Long division- division by a double digit Extend to numbers with 2 decimal places Divide numbers up to 4 digits by a 2 digit number. Show remainders as whole number remainders, fractions, decimals or rounding. 6.25 53.)331.25 −318 132 −106 265 −265 0